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COST EFFICIENT ESTIMATION IN PRESENCE OF NON-RESPONSE AND MEASUREMENT ERRORS USING AUXILIARY VARIABLE

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ABSTRACT

Classes of estimators of population mean which are cost efficient under measurement and non-response errors using auxiliary information are presented. These classes of cost efficient estimators have been proposed when both the errors occurs simultaneously as an alternative to the class of estimators proposed for only non-response by Singh & Kumar (2010) and Singh & Bhushan (2012). The results of the proposed classes are derived. The proposed estimators are put to test against Singh and Kumar (2010) and Singh and Bhushan (2012) estimators under the cost efficiency criteria. The estimators are compared theoretically and empirically.

1. INTRODUCTION

Parameter estimation when measurement errors are present has received considerable attention over the last several decades. In sampling techniques, it is usually assumed that the correct measurements have been made on the characteristics under investigation. This assumption is typically not met, and data has measurement errors such as reporting and tabulation. If ignored, the measurement errors may lead to invalid results. Also, if measurement errors are negligible, then the statistical inferences based on such contaminated data continue to remain valid. On other hand, if the measurement errors are not negligible, the related statistical inferential procedures may not be accurate or may simply be invalid while leading to undesirable, unexpected and unfortunate consequences. Some important references related to the errors of measurement while applying it to the survey data are discussed at length in Cochran (1968), Shalabh (1997), Srivastava & Shalabh (2001) and Singh & Karpe (2008, 2010) while studying different estimation procedures for population mean under measurement errors.

Now, consider a population $U = (U_1, U_2, ...U_N)$ of size N. Let Y be the primary variable and X the auxiliary variable; using simple random sampling(SRS) design, n pairs of observations are collected on X and Y. Let (x_i, y_i) be values that were reported instead of the actual true values (X_i, Y_i), for i^{th} (i=1, 2,...,n) unit, where

$$u_i = y_i - Y_i \tag{1.1}$$

$$v_i = x_i - X_i \tag{1.2}$$

so that u_i and v_i are combined as errors of measurement in y_i and x_i respectively which are random variables with zero mean, and variances σ_u^2, σ_v^2 respectively. Further, we assume that, although X_i's and Y_i's are correlated, u_i's and v_i's are uncorrelated.

Further, currently a lot of attention has been paid to the problem of non-response which is very undesirable though unavoidable feature of sample surveys as it affects the unbiasedness and reliability of the estimates. Most of the time, the information required may not be collected from the selected units in the sample even after various call-backs. Hansen and Hurwitz (1946), in their seminal paper, gave an inventive idea of estimation in presence of non-response using double sampling design based on simple random sampling. They considered the problem of non-response in a mailed questionnaire survey in order to estimate the population mean by drawing a sub-sample from the stratum of non-responding units by direct interview and an estimator was proposed as convex combination of the information available from both the response stratum as well as from the stratum of non-responding units.

It is a very well-known practice among survey statisticians to incorporate some auxiliary variable which closely related to study variable for drawing accurate inferences. When the true mean of the auxiliary variable $\bar{X}$ is known, estimation of the population mean in presence of deterministic non-response has been considered by Singh and Kumar (2008), Khare and Srivastava (1993, 1997), Rao (1986, 1987), Cochran (1977), among others. Further, when such auxiliary information is not available, a two-phase sampling approach is generally used. (Singh and Bhushan, 2012; Singh and Kumar, 2010; Tabassum and Khan, 2004; Okafor and Lee, 2000). Okafor and Lee (2000) proposed using the sample mean $\bar{x}'$ based on a large first phase sample of size n' selected from N units by SRSWOR when the population mean of auxiliary variable $\bar{X}$ is not known. An assumption was made in such studies that the first phase units collected in the sample supplied complete the auxiliary information. This is a standard assumption (Bhushan and Naqvi, 2015; Singh and Kumar, 2010) that complete response is available for the auxiliary variable. Subsequently, a sample of size n is drawn at second phase from the n' $(n < n')$ by simple random sampling design and study variable y is measured on the selected units. Further, suppose n_1 units respond to the survey call and n_2 units do not respond to the survey call out of the n sample units drawn at second phase. So, now we adopt Hansen and Hurwitz (1946) sampling design to select a sub-sample of size r units from n_2 units that were not responding to the survey call so that $r = n_2 / k$ where $k > 1$. We implicitly assume here that these selected r units will respond to the next survey call. An example was cited by Tabassum and Khan (2004) and by Okafor and Lee (2000) to justify the use in practice.

In this study, the setup of deterministic non-response situation as described in Singh and Bhushan (2012), Tabassum and Khan (2004), Okafor and Lee (2000) is used, and entire population U is split into two strata, the respondent stratum denoted by U_1 which has N_1 units that respond on the first call of the second phase while the non-respondent stratum denoted by U_2 with N_2 units which did not responded on the first call but did respond on the second call. Also, denote the first and second phase samples respectively by s and s', such that $s_1 = s \cap U_1$ and $s_2 = s \cap U_2$. Let the second phase sub-sample of s_2 be denoted by s_{2m}, the population parameters by uppercase letters and the sample statistics denoted by lowercase letters.

Okafor and Lee (2000) ratio estimator and revisited by Tabassum and Khan (2004) is

$$T_1 = \frac{\bar{y}^*}{\bar{x}^*}\bar{x}' \tag{1.3}$$

Further, perusing this idea Singh and Kumar (2010) proposed some more estimators given by

$$T_2 = \frac{\bar{y}^*}{\bar{x}'}\bar{x}^* \tag{1.4}$$

$$T_3 = \frac{\bar{y}^*}{\bar{x}}\bar{x}' \tag{1.5}$$

$$T_4 = \frac{\bar{y}^*}{\bar{x}'}\bar{x} \tag{1.6}$$

$$T_{11} = \bar{y}^*\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha} \tag{1.7}$$

$$T_{12} = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)^{\alpha} \tag{1.8}$$

$$T_7 = \bar{y}^*\left(\frac{\bar{x}}{\bar{x}^*}\right)^{\alpha_1}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha_2} \tag{1.9}$$

$$T_8 = \bar{y}^* + d_1\left(\bar{x} - \bar{x}^*\right) + d_2\left(\bar{x}' - \bar{x}\right) \tag{1.10}$$

where $\bar{x}$ is the auxiliary variable mean of n units; $\bar{x}'$ is the auxiliary variable mean of n' units and $\bar{y}^*$ is the primary variable mean with $\bar{x}^* = \frac{n_1}{n}\bar{x}_{(1)} + \frac{n_2}{n}\bar{x}^*_{(2)}$ and $\bar{y}^* = \frac{n_1}{n}\bar{y}_{(1)} + \frac{n_2}{n}\bar{y}^*_{(2)}$ $\left(\text{where } w_1 = \frac{n_1}{n} \text{ and } w_2 = \frac{n_2}{n}\right)$ with $\left(\bar{x}^*_{(2)}, \bar{y}^*_{(2)}\right)$ and $\left(\bar{x}_{(1)}, \bar{y}_{(1)}\right)$ being the sample means of the (x, y) pertaining to sub-sample means based on r units and based on first phase means based on n_1 units respectively; α, α_1, α_2, d_1 and d_2 are the scalars that are suitably determined.

It is important to note that $\bar{y}^*$ is the Hansen Hurwitz estimator; T_1 is the Okafor and Lee (2000) ratio estimator; T_2, T_3, T_4 and T_5 are studied by Singh and Kumar (2010) and found that T_7 performs better that all other alternatives under optimum conditions. Further, T_{11} and T_{12} are included by us to make our study comprehensive. The idea behind construction of T_7 by Singh and Kumar (2010) was to chain T_{11} and T_{12} so as to ensue from maximum gain in efficiency.

The estimators T_1, T_2, T_3, T_4, T_{11} and T_{12} along with some other difference type estimators were generalized by Singh and Bhushan (2012) by

$$t_h = h\left(\bar{y}^*, \bar{x}^*, \bar{x}'\right) \tag{1.11}$$

$$t_H = H\left(\bar{y}^*, \bar{x}, \bar{x}'\right) \tag{1.12}$$

where $h(.)$ and $H(.)$ being bounded functions satisfy the following regularity conditions such that

(i) $h(\mathbf{P}) = \bar{Y}$ and $H(\mathbf{P}) = \bar{Y}$ (1.13)

(ii) first order partial derivative of $h\left(\bar{y}^*, \bar{x}^*, \bar{x}'\right)$ and $H\left(\bar{y}^*, \bar{x}, \bar{x}'\right)$ with respect to $\bar{y}^*$ at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ is unity, that is, $h_0 = 1$ and $H_0 = 1$ (1.14)

(iii) $h_{00} = 0$ and $H_{00} = 0$ (1.15)

(iv) first order partial derivative of $h\left(\bar{y}^*, \bar{x}^*, \bar{x}'\right)$ and $H\left(\bar{y}^*, \bar{x}, \bar{x}'\right)$ with respect to $\bar{x}^*$ and $\bar{x}$ respectively at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ satisfy

$$h_1 = -h_2 \text{ and } H_1 = -H_2 \tag{1.16}$$

(v) $h_{01} = -h_{02}$ and $H_{01} = -H_{02}$ (1.17)

These conditions are similar to the ones in Diana and Tommasi (2003).

Though the idea of Singh and Kumar (2010), motivated by Singh and Kumar (2008), was ingenious in the construction of estimator thereby increasing efficiency but it still incurred extra cost due to "double sampling". The motivation behind this work is that if the auxiliary parameter is unknown and the data on auxiliary variable is available with 100% response, then the approach of Hansen and Hurwitz can be used on the auxiliary sub – sample mean based on S_{2m}, which is a two phase sampling scheme (Lohr, 1999) in contrast to the "double sampling" of Okafor and Lee (2000). Also, Bhushan and Naqvi (2015) investigated families of generalized efficient estimators in the presence of non-response. Bhushan and Pandey (2019a, b, c) provided some efficient procedures for estimation of the population mean under non-response. Bhushan and Pandey (2020) considered the problem of cost efficient estimation in presence of non-response. Bhushan and Pandey (2018, 20, 21) and Bhushan et al. (2020) provided some methods of imputing the missing data.

2. SUGGESTED CLASSES OF ESTIMATORS

Our main objective here is to propose generalised cost-efficient classes of estimators which do not require double in case the auxiliary population mean is unknown. These sampling strategies are compared to the "double sampling" estimators of Singh and Bhushan (2012), Singh and Kumar (2010), Okafor and Lee (2000), Khare and Srivastava (1993, 1995).

In this section, following Diana and Tommasi (2003), we will use the Hansen Hurwitz technique for sub-sampling the non-responders and consider the following classes of estimators:

$$\bar{y}_g^* = \bar{y}^* g\left(\bar{x}^*, \bar{x}\right) \tag{2.1}$$

$$\bar{y}_G^* = G\left(\bar{y}^*, \bar{x}^*, \bar{x}\right) \tag{2.2}$$

where $g(.)$ and $G(.)$ are the bounded functions satisfying the following regularity conditions such that

(i) $g(\mathbf{Q}) = 1$ and $G(\mathbf{P}) = \bar{Y}$ (2.3)

(ii) partial derivative of first order for $G\left(\bar{y}^*, \bar{x}, \bar{x}\right)$ with respect to $\bar{y}^*$ at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ is unity, that is, $G_0 = 1$ (2.4)

(iii) partial derivative of first order for $g\left(\bar{x}^*, \bar{x}\right)$ at $\mathbf{Q} \equiv \left(\bar{X}, \bar{X}\right)$ and of $G\left(\bar{y}^*, \bar{x}, \bar{x}\right)$ at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ with respect to $\bar{x}^*$ and $\bar{x}$ respectively satisfy $g_1 = -g_2$ and $G_1 = -G_2$ (2.5)

(iv) partial derivative of second order for $G\left(\bar{y}^*, \bar{x}, \bar{x}\right)$ at

$\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ with respect to $\left(\bar{y}^*, \bar{x}^*\right)$ and $\left(\bar{y}^*, \bar{x}\right)$ respectively satisfy $G_{01} = -G_{02}$ (2.6)

It may be noted here that $\bar{y}_G^*$ is an extended class of estimator and $\bar{y}_g^*$ is also a subclass of this wider class.

Theorem 2.1:

(i) The bias of the proposed class of estimators is given by

$$Bias\left(\bar{y}_G^*\right) = \left[\frac{(N-n)}{Nn}\left\{\frac{1}{2}\left(G_{11} + G_{22} + 2G_{12}\right)\left(\sigma_x^2 + \sigma_v^2\right)\right\} + \frac{(k-1)W_2}{n}\left\{\sigma_{yx_{(2)}} G_{01} + \frac{\bar{X}^2}{2} C_{x_{(2)}}^2 G_{11}\right\}\right] \tag{2.7}$$

(ii) the MSE's of the proposed class of estimators is given by

$$MSE\left(\bar{y}_G^*\right) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[\left(\sigma_x^2 + \sigma_v^2\right)G_1^2 + 2\sigma_{yx_{(2)}} G_1\right] \tag{2.8}$$

These results are reported up to the first order of approximation.

Corollary 2.2:

(i) The bias of the proposed class of estimators is given by

$$Bias\left(\bar{y}_g^*\right) = \left[\frac{(N-n)}{Nn}\frac{1}{2}\left(g_{11} + g_{22} + 2g_{12}\right)\left(\sigma_x^2 + \sigma_v^2\right) + \frac{(k-1)W_2}{n}\left\{\sigma_{yx_{(2)}} g_1 + \frac{1}{2}\left(\sigma_{x_{(2)}}^2 + \sigma_{v_{(2)}}^2\right) g_2\right\}\right] \tag{2.9}$$

(ii) the MSE's of the proposed class of estimators is given by

$$MSE\left(\bar{y}_g^*\right) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[\bar{Y}^2\left(\sigma_{x_{(2)}}^2 + \sigma_{v_{(2)}}^2\right)g_1^2 + 2\bar{Y}\sigma_{yx_{(2)}} g_1\right] \tag{2.10}$$

These results are reported up to the first order of approximation.

Theorem 2.3: The optimum value of the derivatives g and G are

$$g_{(opt)} = -\frac{\sigma_{yx_{(2)}}}{\bar{Y}\left(\sigma^2_{x_{(2)}} + \sigma^2_{v_{(2)}}\right)} = -d \tag{2.11}$$

$$G_{1(opt)} = -\frac{\sigma_{yx_{(2)}}}{\left(\sigma^2_{x_{(2)}} + \sigma^2_{v_{(2)}}\right)} = -D \tag{2.12}$$

and minimum MSE's are

$$MSE\left(\bar{y}^*_g\right)_{\min} = MSE\left(\bar{y}^*\right) - \frac{(k-1)W_2}{n}\frac{\sigma^2_{yx_{(2)}}}{\left(\sigma^2_{x_{(2)}} + \sigma^2_{v_{(2)}}\right)} \tag{2.13}$$

$$MSE\left(\bar{y}^*_G\right)_{\min} = MSE\left(\bar{y}^*\right) - \frac{(k-1)W_2}{n}\frac{\sigma^2_{yx_{(2)}}}{\left(\sigma^2_{x_{(2)}} + \sigma^2_{v_{(2)}}\right)} \tag{2.14}$$

These are accurate to the first order of approximation.

It may be noted that the same inequality holds for the class $\bar{y}^*_G$ and the minimum MSE of the two classes of estimators is same at that of (2.14). It can be easily appreciated that the Hansen Hurwitz estimator is improved by the proposed generalized estimators under optimality conditions (2.11) and (2.12).

3. OPTIMUM VALUES OF *N* AND *K*

Let us consider the following cost function

$$C = cn + c_1 n_1 + c_2 r \tag{3.1}$$

where

c = per unit cost of obtaining the sample of size n on first attempt

c_1 = per unit cost for processing the respondent data for n_1 units on the first attempt and

c_2 = per unit cost for obtaining data on the r units from n_2 units in the subsample

The values of r and n_1 are not known so we use the expected costs:

$$\mathrm{E}(r)=\frac{W_2 n}{k} \text{ and } \mathrm{E}(n_1)=W_1 n \text{ and then } E(C)=C^*=n\left[c+c_1W_1+\frac{c_2W_2}{k}\right] \quad (3.2)$$

CASE I: Fixed Precision

Theorem 3.1: The optimum values of k and n minimizing the cost for given variance V_0 is

$$k_{opt}=\sqrt{\frac{c_2\left(U_1-W_2U_2\right)}{\left(c+c_1W_1\right)U_2}} \quad (3.1.1)$$

and

$$n_{opt}=\frac{\left\{U_1+\left(k_{opt}-1\right)W_2U_2\right\}}{\left\{V_0+\frac{U_1}{N}\right\}} \quad (3.1.2)$$

respectively. The minimum cost is given by

$$C^*=\frac{1}{V_0}\left[\left(\sigma_y^2+\sigma_u^2\right)+(k_{opt}-1)W_2\left\{\left(\sigma^2_{y_{(2)}}+\sigma^2_{u_{(2)}}\right)+\left(\sigma^2_{x_{(2)}}+\sigma^2_{v_{(2)}}\right)G_1^2+2\sigma_{yx_{(2)}}G_1\right\}\right]\left[c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right] \quad (3.1.3)$$

CASE II: Fixed Cost

Suppose C_0 be the given total cost of the survey and do not include the overhead cost and we wish to determine minimum MSE for a fixed cost such that $C^*<C_0$.

The MSE of $\bar{y}_G^*$ can be expressed as

$$MSE\left(\bar{y}_G^*\right)=\left[\frac{U_1}{n}+\frac{kW_2U_2}{n}-\frac{W_2U_2}{n}-\frac{U_1}{N}\right] \quad (3.3)$$

where $U_1=\left\{\sigma_y^2+\sigma_u^2\right\}$ and $U_2=\left\{\left(\sigma^2_{y_{(2)}}+\sigma^2_{u_{(2)}}\right)+\left(\sigma^2_{x_{(2)}}+\sigma^2_{v_{(2)}}\right)G_1^2+2\sigma_{yx_{(2)}}G_1\right\}$

Theorem 3.2: The optimum values of k and n minimizing the $MSE(\bar{y}_G^*)$ for the given cost ($C^* < C_0$) are given by

$$k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2 U_2\right)}{\left(c + c_1 W_1\right) U_2}} \tag{3.2.1}$$

and

$$n_{opt} = \frac{C_0}{\left\{c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right\}} \tag{3.2.2}$$

respectively. The minimum mean square error for $\bar{y}_G^*$ at given cost ($C^* < C_0$) is

$$C^* = \frac{1}{C_0}\left[\left(\sigma_y^2 + \sigma_u^2\right) + (k_{opt} - 1)W_2\left\{\left(\sigma_{y_{(2)}}^2 + \sigma_{u_{(2)}}^2\right) + \left(\sigma_{x_{(2)}}^2 + \sigma_{v_{(2)}}^2\right)G_1^2 + 2\sigma_{yx_{(2)}} G_1\right\}\right]\left[c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right] \tag{3.2.3}$$

A similar result for $\bar{y}_g^*$ can also be easily obtained.

4. AN EMPIRICAL STUDY

The data pertaining to physical growth of upper socio- economic group of 98 school going children of Varanasi under an ICMR study, see Khare and Sinha (2007). The first 24 children (i.e. 25%) units have been chosen as non-responding units. The parameters values related to the auxiliary character x (chest circumference of the children in cm) and study character y (weight of children in kg), have been reported as given below:

$\bar{X} = 55.8611$; $\bar{Y} = 19.4968$; $\sigma_x = 3.2735$; $\sigma_y = 3.0435$; $\sigma_{x_{(2)}} = 2.5137$; $\sigma_{y_{(2)}} = 02.3552$;

$\sigma_{yx} = 8.428611$; $\sigma_{yx_{(2)}} = 4.315874$, $c = 15$, $c_1 = 22$, $c_2 = 60$, $c' = 12$, $n = 37$, $n' = 70$, $N_1 = 70$, $N_2 = 28$, $V_0 = 0.45$, $C_0 = 1100$.

The problem under consideration is the estimation of mean weight of the children aged between 6 and 7 years using chest circumference of children as the auxiliary character.

Table 4.1: Expected cost and cost efficiency

Estimator	*% ME*	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
	0%	1.6445	24.5947	-	1148.1830	1.0000
	1%	1.6445	27.0205	-	1098.6300	0.9900
$\bar{y}^*$	5%	1.6445	19.6512	-	1205.5930	0.9524
	10%	1.6445	31.0631	-	1263.0002	0.9091
	15%	1.6445	32.4751	-	1320.4110	0.8695
	20%	1.6445	33.8871	-	1377.820	0.8333
	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
	0%	2.2063	28.5272	-	1092.0424	1.0505
	1%	2.1954	27.2958	-	1046.697	1.0392
t_1	5%	2.1552	29.9514	-	1152.383	0.9963
	10%	2.1115	31.3730	-	1211.666	0.9476
	15%	2.0734	32.7924	-	1270.792	0.9035
	20%	2.0401	34.2100	-	1329.814	0.8634
	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
	0%	0.4998	17.2517	-	1065.674	1.0774
	1%	0.5051	16.6528	-	1023.317	1.0629
t_2	5%	0.5258	18.8739	-	1137.566	1.0093
	10%	0.5499	204756	-	1208.164	0.9503
	15%	0.5725	22.0600	-	1277.676	0.8986
	20%	0.5937	23.6297	-	1346.261	0.8529
	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
	0%	2.5137	26.6865	-	1067.6250	1.0754
t_3	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705
	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
t_4	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705

Estimator	*% ME*	k_{opt}	n	n'	*Cost*	*Cost Efficiency*
T_1	0%	1.4912	21.3565	37.6974	1340.953	0.8562
	1%	1.4942	21.7521	37.9019	1359.415	0.8446
	5%	1.5053	23.3229	38.6823	1432.356	0.8016
	10%	1.5171	25.2623	39.5806	1521.664	0.7545
	15%	1.5271	27.1779	40.4025	1609.122	0.7135
	20%	1.5357	29.0724	41.1557	1697.924	0.6774
	% ME	k_{opt}	n	n'	*Cost*	*Cost Efficiency*
T_3	0%	1.0628	19.9108	38.057	1366.704	0.8401
	1%	1.07219	20.3264	38.2659	1385.657	0.8286
	5%	1.1072	21.9708	39.0576	1460.283	0.7863
	10%	1.1461	23.9912	39.9628	1551.187	0.7402
	15%	1.1805	25.9781	40.7858	1639.805	0.7002
	20%	1.2111	27.9360	41.5365	1726.433	0.7764
	% ME	k_{opt}	n	n'	*Cost*	*Cost Efficiency*
T_5	0%	1.0427	13.6709	41.2635	1123.736	1.0217
	1%	1.0677	14.3120	41.6246	1152.685	0.9961
	5%	1.1542	16.7899	42.8599	1262.733	0.9093
	10%	1.2400	19.7347	44.0244	1390.024	0.8260
	15%	1.3087	225510	44.5879	1508.540	0.7611
	20%	1.8635	25.2655	45.4202	1619.988	0.7087
	% ME	k_{opt}	n	n'	*Cost*	*Cost Efficiency*
T_7	0%	1.2047	13.1686	39.0374	1048.192	1.0377
	1%	1.2259	14.5716	41.5669	1137.136	1.0097
	5%	1.2947	17.1215	42.7793	1252.140	0.9169
	10%	1.3561	20.0676	43.9132	1382.226	0.8307
	15%	1.4008	22.8108	44.7573	1501.143	0.7649
	20%	1.4346	25.3957	45.3982	1611.594	0.7124
	% ME	k_{opt}	n	n'	*Cost*	*Cost Efficiency*
T_9	0%	1.2046	13.9002	41.2062	1106.424	1.0340
	1%	1.2277	14.6684	41.5553	1140.9660	1.0063
	5%	1.2958	17.2090	42.7512	1255.408	0.9146
	10%	1.3567	20.1461	43.87050	1384.959	0.8290
	15%	1.4011	22.8820	44.7041	1503.458	0.7637
	20%	1.4348	25.4608	45.3371	1613.567	0.7116
Estimator	*% ME*	k_{opt}	n	n'	*Cost*	*Cost Efficiency*
T_{10}	0%	0.6510	10.2888	38.6349	1026.685	1.0595
	1%	0.6846	11.6723	41.2631	1120.574	1.0246
	5%	0.7974	14.6572	42.7939	1252.998	0.9163
	10%	0.9053	17.9767	44.1272	1395.733	0.8226
	15%	0.9899	209887	45.0692	1522.139	0.7573
	20%	1.0586	23.7788	45.7578	1637.226	0.7013

	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
T_8	0%	1.2047	13.1686	39.0374	1048.192	1.0377
	1%	1.2259	14.5716	41.5669	1137.1360	1.0097
	5%	1.2947	17.1215	42.7793	1252.140	0.9169
	10%	1.3562	200676	43.9132	1382.226	0.8307
	15%	1.4008	22.8108	44.7573	1501.143	0.7649
	20%	1.4346	25.3957	45.3982	1611.594	0.7125
	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
$\overline{y}_g^*$	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705
↓ Estimators				$1/k$		
	% ME	k_{opt}	*n*	*n′*	*Cost*	*Cost Efficiency*
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705

Table 4.2: PRE & MSE under measurement error & non-response

↓ Estimators			$1/k$		
Estimators	*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
$\overline{y}^*$	0%	0.2904(100)	0.3304(100)	0.3705(100)	0.4105(100)
	1%	0.2933(99)	0.3337(99)	0.3742(99)	0.4146(99)
	5%	0.3049(95)	0.3469(95)	0.3889(95)	0.4310(95)
	10%	0.3194(91)	0.3635(91)	0.4075(91)	0.4516(91)
	15%	0.3339(87)	0.3799(87)	0.4260(87)	0.4721(87)
	20%	0.3485(83)	0.3965(83)	0.4446(83)	0.4926(83)
	↓ME %	*1/2*	*1/3*	*1/4*	*1/5*
t_1	0%	0.2742(106)	0.2980(111)	0.3219(115)	0.3457(119)
	1%	0.2771(105)	0.3014(110)	0.3257(114)	0.3501(117)
	5%	0.2889(100)	0.3151(105)	0.3413(109)	0.3674(112)
	10%	0.3038(96)	0.3322(99)	0.3606(103)	0.3890(106)
	15%	0.3186(91)	0.3493(95)	0.3799(98)	0.4106(100)
	20%	0.3334(87)	0.3664(90)	0.3993(93)	0.4323(95)

	↓*ME* %	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.4247(68)	0.5990(55)	0.7734(48)	0.9477(43)
t_2	1%	0.4276(68)	0.6024(55)	0.7772(48)	0.9520(43)
	5%	0.4395(66)	0.6161(54)	0.7927(47)	0.9693(42)
	10%	0.4543(64)	0.6332(52)	0.8121(46)	0.9909(41)
	15%	0.4691(62)	0.6502(51)	0.8314(45)	1.0126(41)
	20%	0.4839(60)	0.6673(50)	0.8508(44)	1.034(40)
	↓*ME* %	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
t_3	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)
	↓*ME* %	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
t_4	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)
	↓*ME* %	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2159(135)	0.2397(138)	0.2636(141)	0.2874(143)
T_{10}	1%	0.2189(133)	0.2432(136)	0.2676(138)	0.2919(141)
	5%	0.2315(125)	0.2576(128)	0.2838(131)	0.3099(132)
	10%	02471(118)	0.2755(120)	0.3039(122)	0.3323(124)
	15%	0.2627(110)	0.2934(113)	0.3241(114)	0.3548(116)
	20%	0.2784(104)	0.3113(106)	0.3443(108)	0.3773(109)
	↓*ME* %	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2321(125)	0.2721(121)	0.3121(119)	0.3522(117)
T_3	1%	0.2351(124)	0.2756(120)	0.3160(117)	0.3564(115)
	5%	0.2474(117)	0.2894(114)	0.3315(112)	0.3735(110)
	10%	0.2628(111)	0.3068(108)	0.3508(106)	0.3949(104)
	15%	0.2781(104)	0.3241(102)	0.3702(100)	0.4162(99)
	20%	0.2935(99)	0.3415(97)	0.3895(95)	0.4376(94)
	↓*ME* %	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.1908(152)	0.2146(154)	0.2385(155)	0.2623(156)
T_5	1%	0.1944(149)	0.2187(151)	0.2430(152)	0.2673(154)
	5%	0.2089(139)	0.2350(141)	0.2612(142)	0.2873(143)
	10%	0.2270(128)	0.2555(129)	0.2839(130)	0.3123(131)
	15%	0.2452(118)	0.2758(120)	0.3065(121)	0.3372(122)
	20%	0.2633(110)	0.2962(111)	0.3292(112)	0.3622(111)

	↓*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
T_7	1%	0.1886(154)	0.2080(159)	0.2273(163)	0.2467(166)
	5%	0.2042(142)	0.2260(146)	0.2477(150)	0.2695(152)
	10%	0.2233(130)	0.2480(133)	0.2727(136)	0.2974(138)
	15%	0.2419(120)	0.2695(123)	0.2971(125)	0.3246(126)
	20%	0.2603(111)	0.2906(114)	0.3209(115)	0.3513(117)
	↓*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.1851(157)	0.2040(162)	0.2229(166)	0.2418(170)
	1%	0.1891(154)	0.2086(158)	0.2281(162)	0.2475(166)
T_9	5%	0.2047(142)	0.2266(146)	0.2484(149)	0.2703(152)
	10%	0.2238(130)	0.2486(133)	0.2734(136)	0.2982(138)
	15%	0.2424(120)	0.2701(122)	0.2977(126)	0.3254(126)
	20%	0.2607(111)	0.2912(113)	0.3216(115)	0.3519(116)
	↓*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2059(141)	0.2460(134)	0.2860(130)	0.3260(126)
	1%	0.2096(139)	0.2501(132)	0.2905(128)	0.3309(124)
T_8	5%	0.2245(129)	0.2665(124)	0.3085(120)	0.3506(117)
	10%	0.2426(120)	0.2867(115)	0.3307(112)	0.3748(110)
	15%	0.2605(111)	0.3065(108)	0.3526(105)	0.3986(103)
	20%	0.2781(104)	0.3261(101)	0.3742(99)	0.4222(97)
	↓*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
	1%	0.1886(154)	0.2080(159)	0.2273(163)	0.2467(166)
$\overline{y}_g^*$	5%	0.2042(142)	0.2259(146)	0.2477(150)	0.2695(152)
	10%	0.2233(130)	0.2479(133)	0.2727(136)	0.2974(138)
	15%	0.2419(120)	0.2695(123)	0.2971(125)	0.3246(126)
	20%	0.2603(111)	0.2906(113)	0.3209(115)	0.3513(117)
	↓*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)
	↓*ME %*	*1/2*	*1/3*	*1/4*	*1/5*
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
$\overline{y}_G$	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)

Table 4.3: MSE (PRE) without measurement error

Estimators	*1/2*	*1/3*	*1/4*	*1/5*
$\bar{y}^*$	0.2904(100)	0.3304(100)	0.3705(100)	0.4105(100)
t_1	0.2742(106)	0.2980(110)	0.3219(114)	0.3457(118)
t_2	0.4247(68)	0.5990(55)	0.7733(48)	0.9477(43)
t_3	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)
t_4	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)
T_1	0.2159(134)	0.2397(138)	0.2636(141)	0.2874(143)
T_3	0.2320(125)	0.2721(121)	0.3121(119)	0.3522(117)
T_5	0.1908(152)	0.2146(154)	0.2385(155)	0.2623(156)
T_7	0.1846(157)	0.2034(162)	0.2221(167)	0.2409(170)
T_9	0.1851(157)	0.2040(162)	0.2229(166)	0.2418(170)
T_{10}	0.2059(141)	0.2459(134)	0.2859(130)	0.3260(126)
T_8	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
T_g	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
T_G	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
$\bar{y}_g^*$	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)
$\bar{y}_G^*$	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)

Table 4.4: Optimum MSE(PRE) at different label of measurement error

Estimators →	*MSE(PRE)*					
Per cent of ME →	*0%*	*1%*	*5%*	*10%*	*15%*	*20%*
$\bar{y}^*$	0.3758(100)	0.3795(99)	0.3946(95)	0.4133(91)	0.4321(87)	0.4509(83)
t_1	0.3577(105)	0.3616(104)	0.3771(100)	0.3965(95)	0.4159(90)	0.4352(86)
t_2	0.3488(108)	0.3535(106)	0.3723(101)	0.3954(95)	0.4181(90)	0.4406(85)
t_3	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.4115(91)	0.4317(87)
t_4	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.4115(91)	0.4317(87)
T_1	0.4388(86)	0.4449(84)	0.4688(80)	0.4979(75)	0.5266(71)	0.5547(68)
T_3	0.4473(84)	0.4535(83)	0.4779(79)	0.5077(74)	0.5367(70)	0.5650(67)
T_5	0.3678(102)	0.3772(100)	0.4132(91)	0.4549(83)	0.4937(76)	0.5302(71)
T_7	0.3621(104)	0.3722(101)	0.4098(92)	0.4524(83)	0.4913(76)	0.5274(71)
T_9	0.3636(103)	0.3736(101)	0.4110 (91)	0.4534(83)	0.4921(76)	0.5282(71)
T_{10}	0.3546(106)	0.3667(102)	0.4101(92)	0.4568(82)	0.4982(75)	0.5358(70)
T_8	0.3621(104)	0.3721(101)	0.4098(92)	0.4524(83)	0.4913(76)	0.5274(71)
$\bar{y}_g^*$	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.411591)	0.4317(87)
$\bar{y}_G^*$	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.411591)	0.4317(87)

5. CONCLUDING REMARKS

1. The expressions (2.8) and (2.10) regarding the MSE's of estimators reflect that the measurement errors have magnified the MSE's of these estimators and thereby reducing the efficiency.

2. The expressions (2.8) and (2.10) regarding MSE can be fragmented into four constituents owing to non-response and measurement errors are stated below:

   $MSE = \mathrm{A} + \mathrm{B} + \mathrm{X} + \Delta$

   where A = MSE Component due to sampling without non-response and measurement errors,

   B = MSE Component due to sampling without non-response and with measurement error,

   X = MSE Component due to sampling without with non-response and measurement error, and

   Δ = MSE Component due to sampling with without non-response and measurement error. For Example: Consider the expression of MSE of $\bar{y}_G^*$ given by

$$MSE(\bar{y}_G^*) = \underbrace{\frac{1}{n}\sigma_y^2}_{\mathrm{A}} + \underbrace{\frac{1}{n}\sigma_u^2}_{\mathrm{B}} + \underbrace{\frac{W_2(k-1)}{n}\left(\sigma_{y_{(2)}}^2 + G_1^2\sigma_{x_{(2)}}^2 + 2G_1\sigma_{xy_{(2)}}\right)}_{\mathrm{X}} + \underbrace{\frac{W_2(k-1)}{n}\left(\sigma_{u_{(2)}}^2 + G_1^2\sigma_{v_{(2)}}^2\right)}_{\Delta}$$

3. The results and expression of the MSE's of various conventional estimators under non – response can be easily derived from the general expression of $MSE(\bar{y}_G^*)$ when measurement errors are absent. The results provide a more application oriented, pragmatic and general approach for the estimation of mean if both non response and measurement errors are present. As an example, if we set $u_i = 0 = v_i$, for each i, so that $\sigma_u^2 = \sigma_v^2 = \sigma_{u_2}^2 = \sigma_{v_2}^2 = 0$ and we get

$$MSE(\bar{y}_G^*) = \underbrace{\frac{1}{n}\sigma_y^2}_{A} + \underbrace{\frac{W_2(k-1)}{n}\left(\sigma_{y_{(2)}}^2 + G_1^2\sigma_{x_{(2)}}^2 + 2G_1\sigma_{xy_{(2)}}\right)}_{X}$$

such that only components A and X will remain in the expression. Also, this expression for MSE is same as provided by Bhushan & Naqvi (2015) while considering $\bar{y}_G^*$.

4. The results and expressions of the optimum characterizing value of the scalars involved and the corresponding minimum MSE's of various conventional estimators under non – response can be easily derived from the general expressions of optimum characterizing value of the proposed derivatives involved and the corresponding minimum MSE's when measurement errors are absent.

   As an example, if we set $u_i = 0 = v_i$, for each i, so that $\sigma_u^2 = \sigma_v^2 = \sigma_{u_2}^2 = \sigma_{v_2}^2 = 0$ and we get optimum value of the derivative G_1 as

   $$G_1 = -\sigma_{xy_{(2)}} / \sigma_{x_{(2)}}^2$$

   and the minimum value of the mean square error of $\bar{y}_G^*$ can be evaluated from

   $$MSE(\bar{y}_G^*)_{\min} = \frac{1}{n}\sigma_y^2 + \frac{W_2(k-1)}{n}\left(1-\rho_{(2)}^2\right)\sigma_{y_{(2)}}^2$$

   Also, this expression regarding the minimum value of MSE is same as provided by Bhushan & Naqvi (2015) while considering $\bar{y}_G^*$.

5. There is no denying that the measurement errors have adversely affects the estimators but the considered cost-efficient estimators $\bar{y}_g^*$ and $\bar{y}_G^*$ outperform the double sampling estimators. This can be easily realized by following empirical results tabulated in the table 4.1 wherein the estimator $\bar{y}_G^*$ exploited the auxiliary information in an optimal manner and outdone all the remaining double sampling estimators when the cost was also considered. An investigation into the expected cost reveals

that the estimators $\bar{y}_g^*$ and $\bar{y}_G^*$ is relatively cheaper to implement than all the other conventional double sampling estimators like by Okafor & Lee (2000), Tabassum & Khan (2004) and Singh & Kumar (2010) when considered under optimum conditions.

6. The double sampling estimators were severely affected by the measurement errors in terms of cost efficiency even under the optimal conditions in comparision to the considered cost efficient estimators. These observations are more evident from the perusal of empirical results from table 4.1 where the loss of cost efficiency was 32% for the double sampling estimator T_G in comparison to our cost-efficient estimator $\bar{y}_G^*$ lost only 20% cost efficiency when auxiliary information was not optimally utilized.

7. The double sampling estimators were severely affected by the measurement errors in terms of efficiency even under the optimal conditions in comparision to the considered cost efficient estimators. For instance, if we consider the empirical results reported in table 4.2 where the loss of efficiency was 53% for the double sampling estimator T_G with 1/5 sub-sampling fraction in comparison to our cost efficient estimator $\bar{y}_G^*$ lost only 23% cost efficiency when auxiliary information was optimally utilized.

8. The per cent relative efficiencies of various conventional estimators w. r. t. $\bar{y}*$ were reported with optimum sub-sampling fraction $1/k$ are given in table 4.4. The optimal choice of estimator is $\bar{y}_g^*$ and $\bar{y}_G^*$ within the class of all considered estimators at various measurement error levels. Further, the performance of the estimators decreases with the increasing level of measurement error. The cost efficient estimators retain more than 100% efficiency even at 5% level of measurement error while the double sampling estimators retain more than 100% efficiency only till 1% level of measurement error.

9. The optimal choice of estimator in terms of efficiency (not cost efficiency), is $\bar{y}_g^*$ and $\bar{y}_G^*$ at all levels of measurement error. The table 4.4 also demonstrates that the standard results of non-response estimators can be obtained when there is no measurement error. The comment 5 reiterated here that estimators $\bar{y}_g^*$ and $\bar{y}_G^*$ are still the most cost-efficient estimators.

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